

The field measurement method

The method described as been set up by the Statistics Division of FAO.

A traditional method of measurement or areas in agricultural statistics consists of identifying the boundaries of a field to be measured by use of sight poles and taking compass bearings and measuring the length of each side of a so obtained polygon.

The data obtained are then computed on a programmed pocket calculator. Using calculators enable to avoid the possible errors in the more classical method of sketching the field. Errors in plotting the sketch, errors in measuring the areas form he sketch and in particular, errors in applying the scale factor are eliminated with the use of the calculator.

The use of the calculators permits the application of methods of distributing the closure error to all vertices, which is superior to the hand method of handling the closure error. The most important advantage of the calculator is the possibility to use it directly in the field when measurement are made as the closure error can be evaluated directly on the spot and in case of too large an error the measurement can be repeated.

Calculation of the area of a polygon

Let a polygon with n sides be defined by

$$a_i, \theta_i \quad i= 1, 2, \dots, n$$

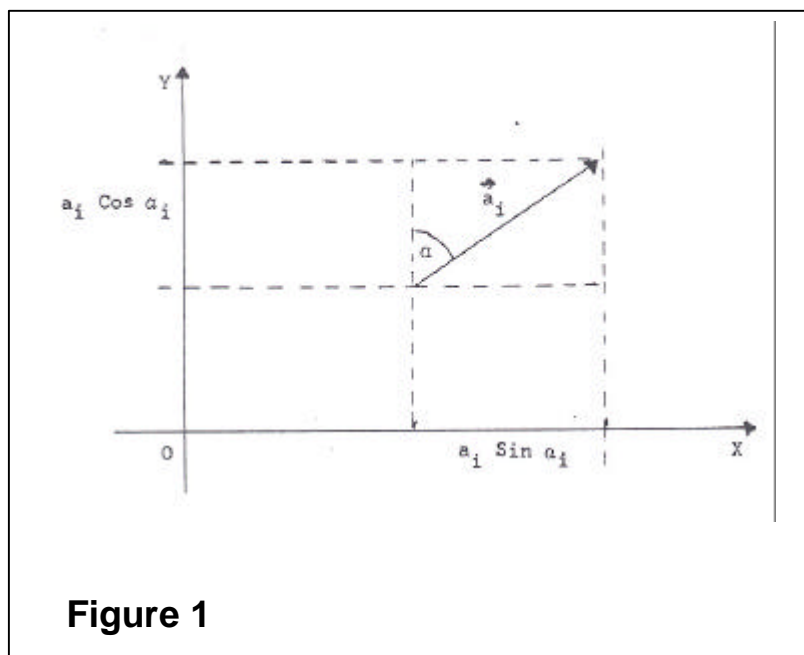
where a_i is the length of the side i and θ_i is the angle this side forms with North measured in clockwise direction.

Denote with \vec{a}_i the vector which represents the side i in a two dimensional space XOY in which Y-axis coincides with the North.

The horizontal and vertical projections of the vector \vec{a}_i (see figure 1) are respectively:

$$a_i \sin \theta_i$$

$$a_i \cos \theta_i$$



Define vectors

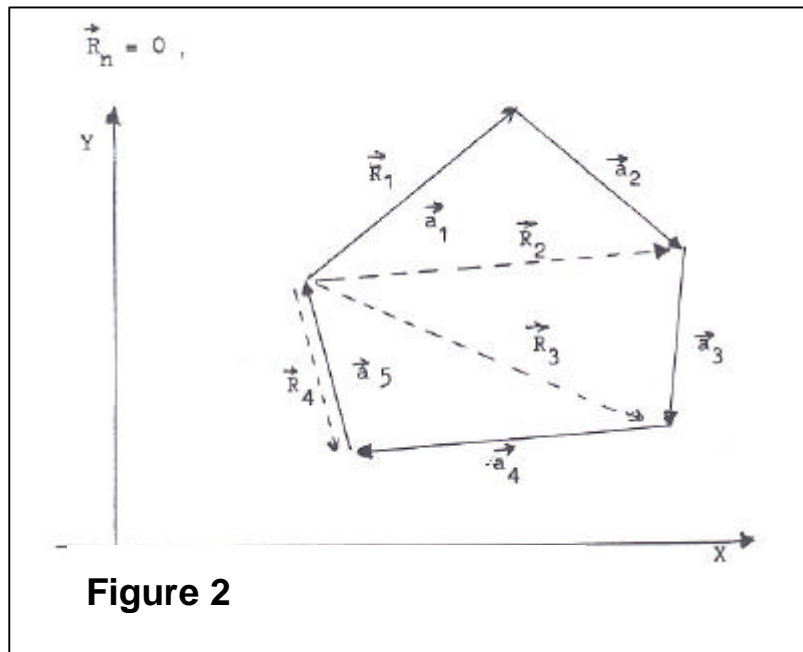
$$(1) \vec{R}_i = \sum_{j=1}^i a_j \quad i=1,2, \dots, n$$

Their horizontal and vertical projections will be respectively:

$$(2) X_i = \sum_{j=1}^i a_j \sin \alpha_j$$

$$(3) Y_i = \sum_{j=1}^i a_j \cos \alpha_j$$

If the polygon is closed, then:



The area of a triangle formed by two vectors which start from the same point can be calculated as a function of their horizontal and vertical projections.

Thus, the area of the triangle between vectors R_1 and R_2 (see figure 2) is given by:

$$A_1 = \frac{1}{2} (X_2 Y_1 - X_1 Y_2)$$

It should be noted that this area will have a positive value if the vector R_1 precedes the vector R_2 (looking clockwise), otherwise it will be negative.

The area of the whole polygon calculated as a sum of areas of triangles, each formed by the two consecutive vector R_i , will be:

$$A = \frac{1}{2} \sum_{i=1}^{n-2} (X_{i+1} Y_i - X_i Y_{i+1})$$

where X_i and Y_i are given by (2) and (3).

Closure error and corrected area of a polygon

In practice the polygon defined by the data which are collected in the field will never close. In this case

$$R_n \neq 0$$

The length of the vector R_n

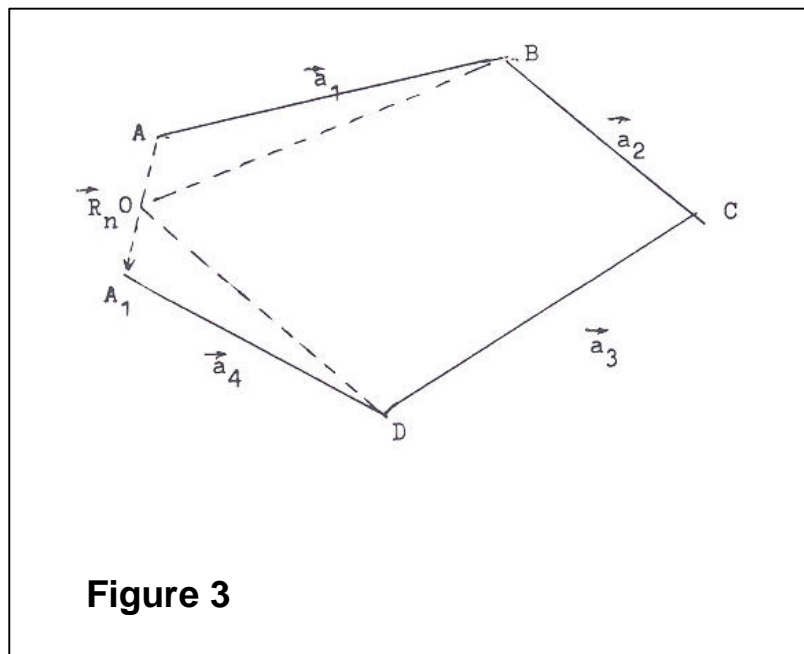
$$R_n = \sqrt{X_n^2 + Y_n^2}$$

can be used as a measure of error. The normal practice is, however, to express the closure error as percent of the perimeter of the polygon:

$$C = \left(\frac{R_n}{\sum_{i=1}^n a_i} \right) \times 100$$

If the closure error is below a certain value, say 2% or 3%, the error may be considered as acceptable. The polygon can be closed in different ways and the area of a so closed polygon calculated.

Closing the polygon: Closure from the mid-point



The closed polygon OBCDO is obtained by connecting the mid-point O between end points A and A₁ with the ends of the first and the last side of the open polygon, that is with points B and D.

Define new vectors R_i':

$$R_i' = R_i - \frac{1}{2} R_n \quad i= 1,2, \dots, n-1$$

with projections

$$(4) \quad X_i' = X_i - \frac{1}{2} X_n$$

$$(5) \quad Y_i' = Y_i - \frac{1}{2} Y_n$$

Then, the area of the closed polygon will be:

$$A = \frac{1}{2} \sum_{i=1}^{n-2} (X'_{i+1} Y'_i - X'_i Y'_{i+1})$$

or after substituting X'_i and Y'_i from (4) and (5)

$$A = \frac{1}{2} \sum_{i=1}^{n-2} (X_{i+1} Y_i - X_i Y_{i+1}) + (Y_n / 4)(X_1 - X_{n-1}) - (X_n / 4)(Y_1 - Y_{n-1})$$

where X_i and Y_i are defined by (2) and (3).